

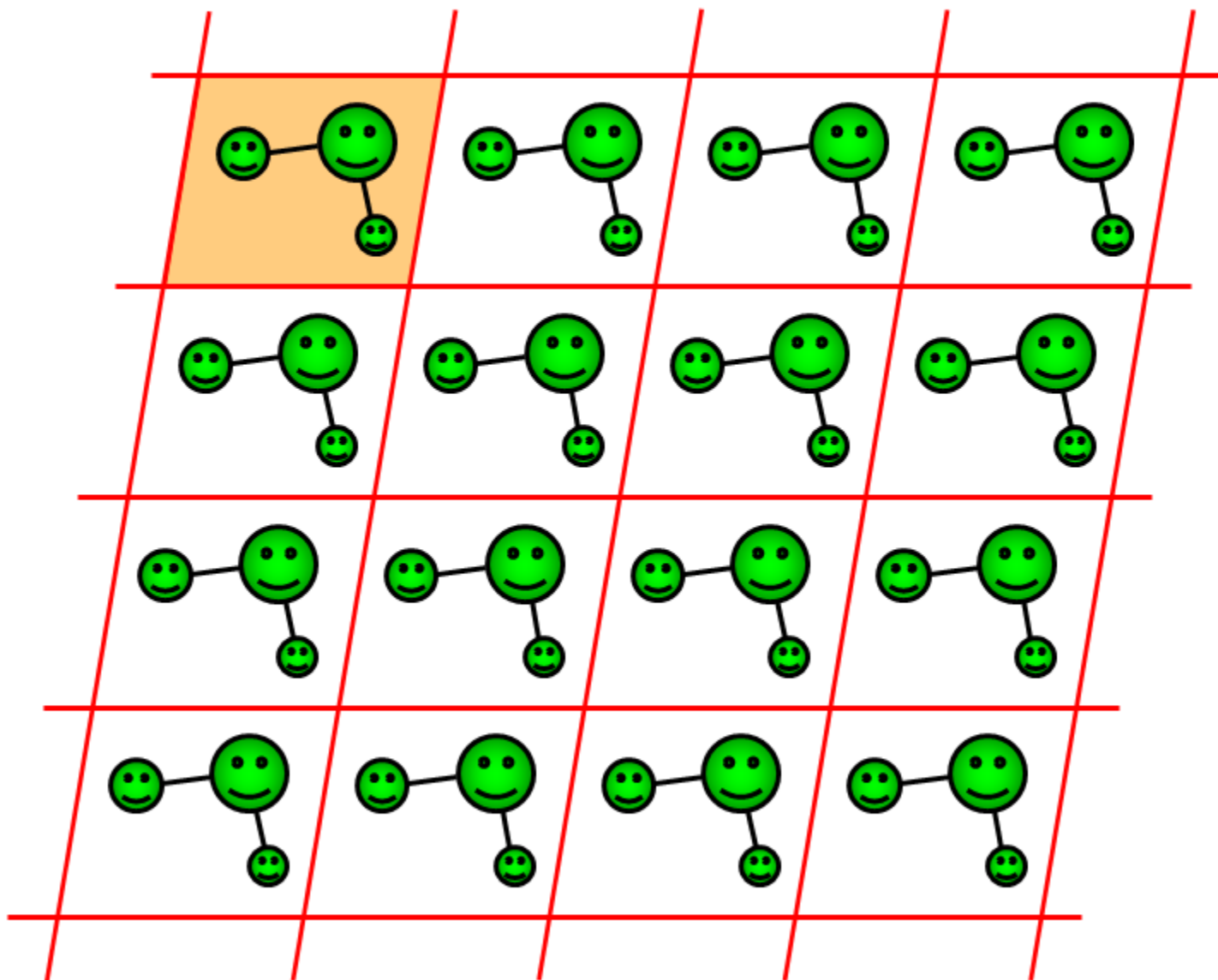
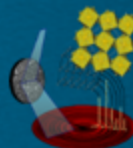


Figure 1.1. Illustration of a two-dimensional lattice with one unit cell shaded in orange.

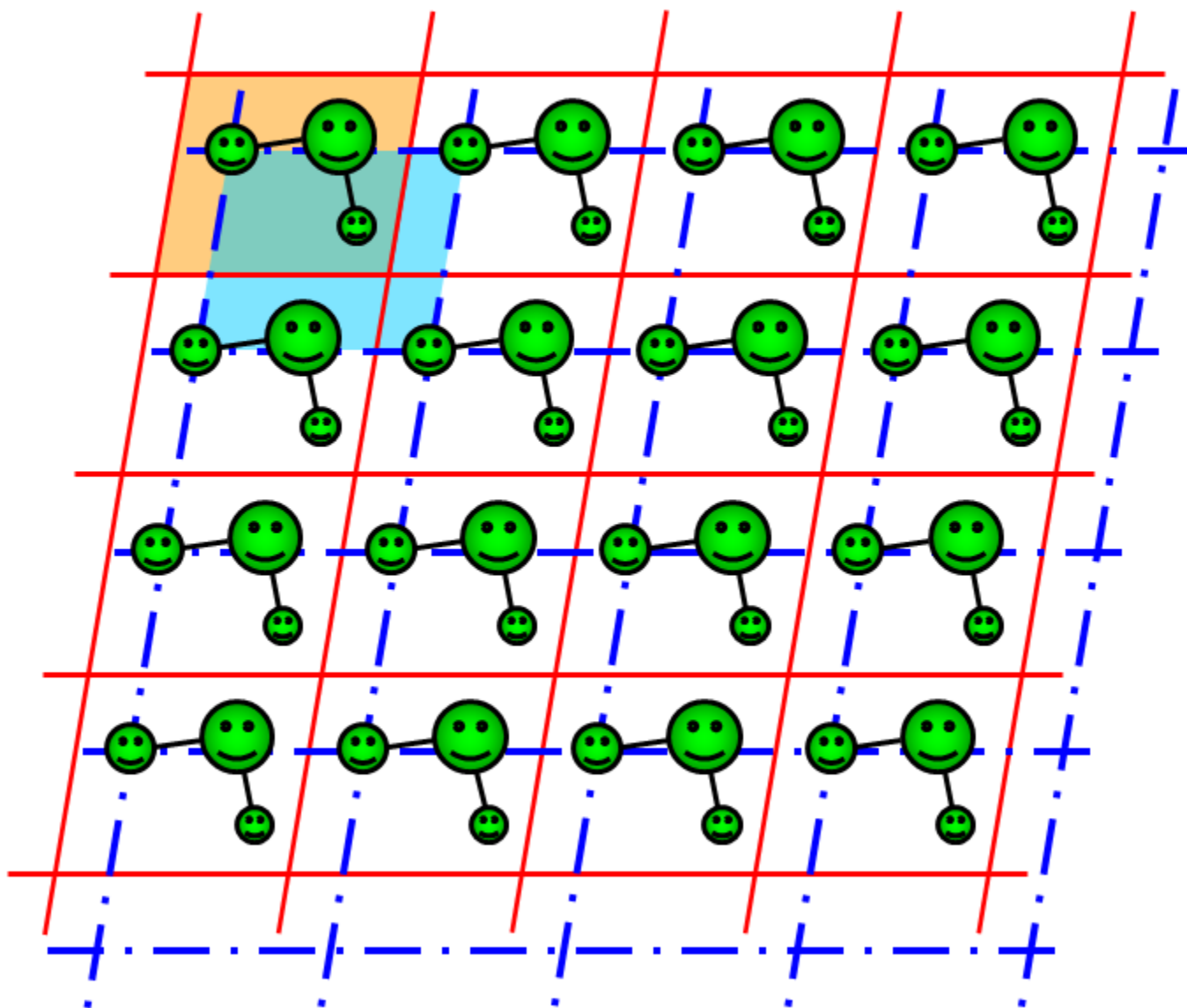
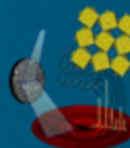


Figure 1.2. Illustration of an arbitrary origin of a lattice. The original and the alternative lattices are shown using *solid* and *dash-dotted* lines, respectively. The unit cells (shaded in *orange* and *blue*) have identical shapes.

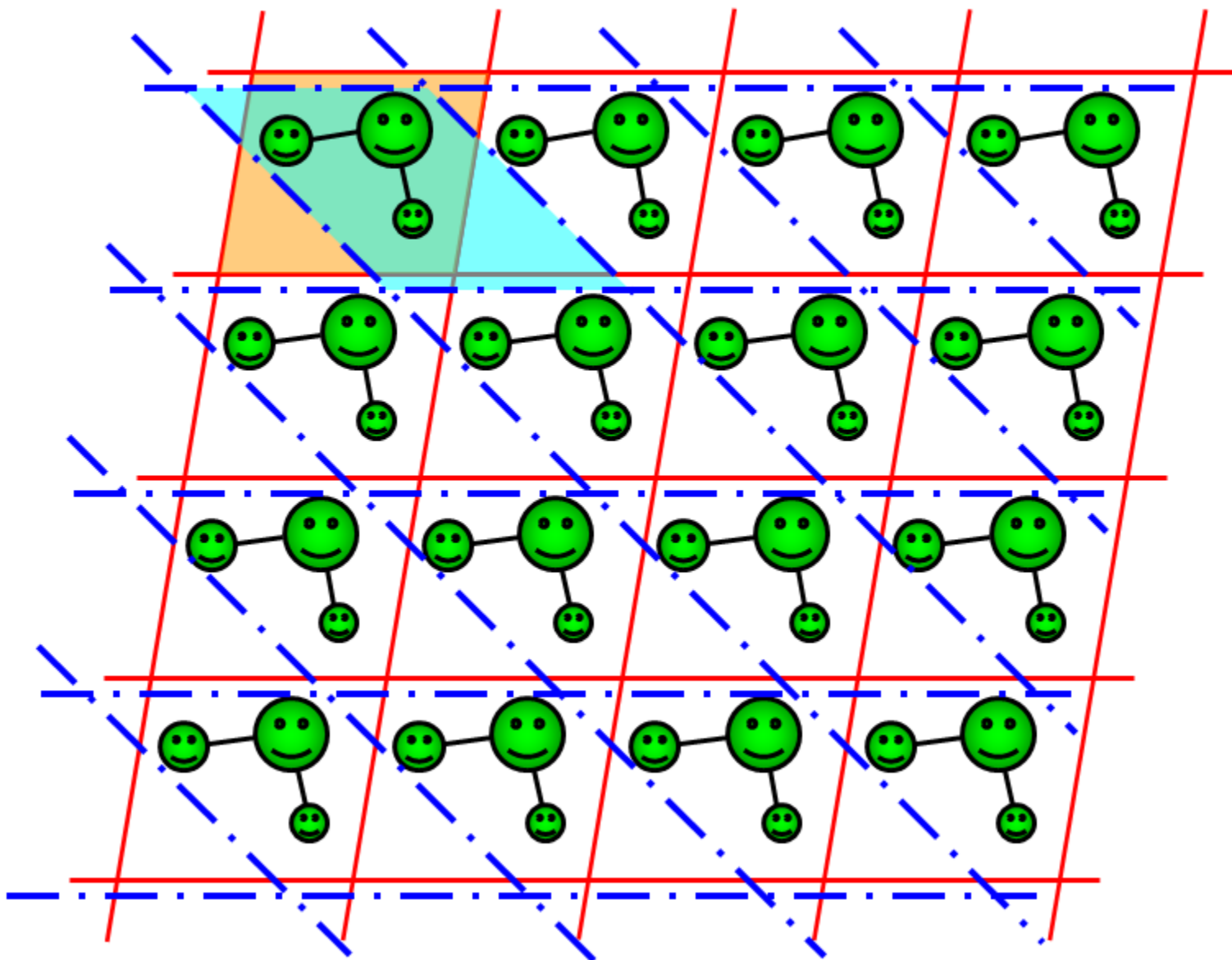
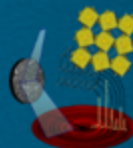
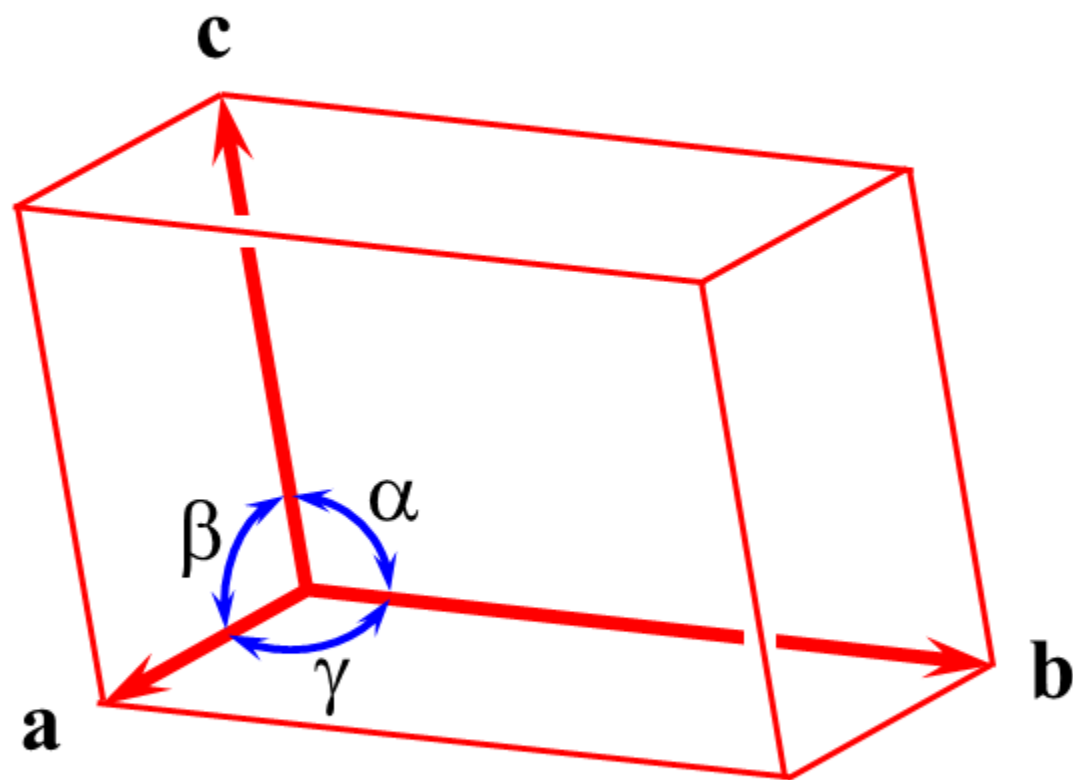
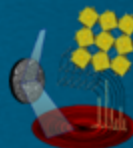


Figure 1.3. Illustration of an arbitrary unit cell of a lattice. The original and the alternative lattices are shown using *solid* and *dash-dotted lines*, respectively. The unit cells (shaded in *orange* and *blue*) have different shapes but their areas (or volumes in three dimensions) and contents remain identical.



$a, b, c, \alpha, \beta, \gamma$

The units of the unit cell edges are:

- Angströms (Å) – $1 \text{ Å} = 10^{-10} \text{ m} = 10^{-8} \text{ cm}$, or
- nanometers (nm) – $1 \text{ nm} = 10^{-9} \text{ m}$, or
- picometers (pm) – $1 \text{ pm} = 10^{-12} \text{ m}$,

and the angles between vectors are:

- degrees (°)

Eq. 1.1. $\mathbf{q} = u\mathbf{a} + v\mathbf{b} + w\mathbf{c}$

Figure 1.4. Unit cell in three dimensions.

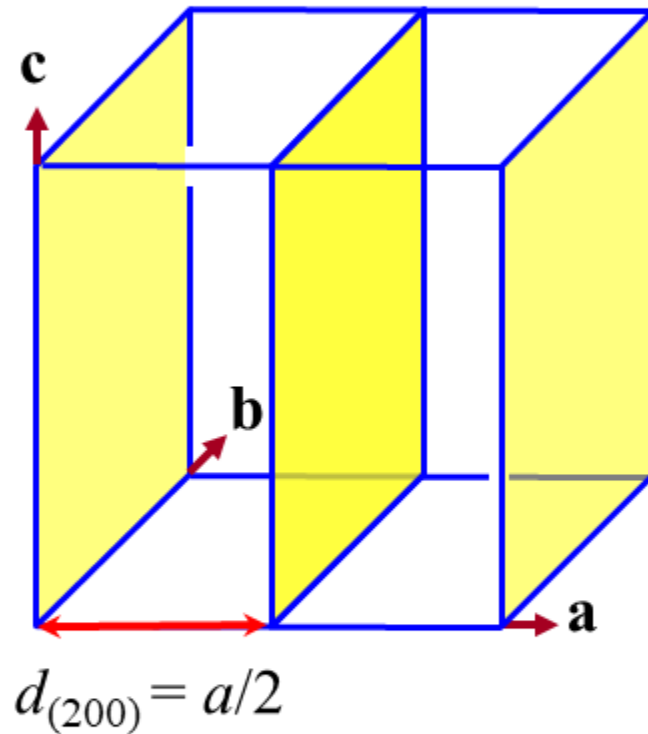
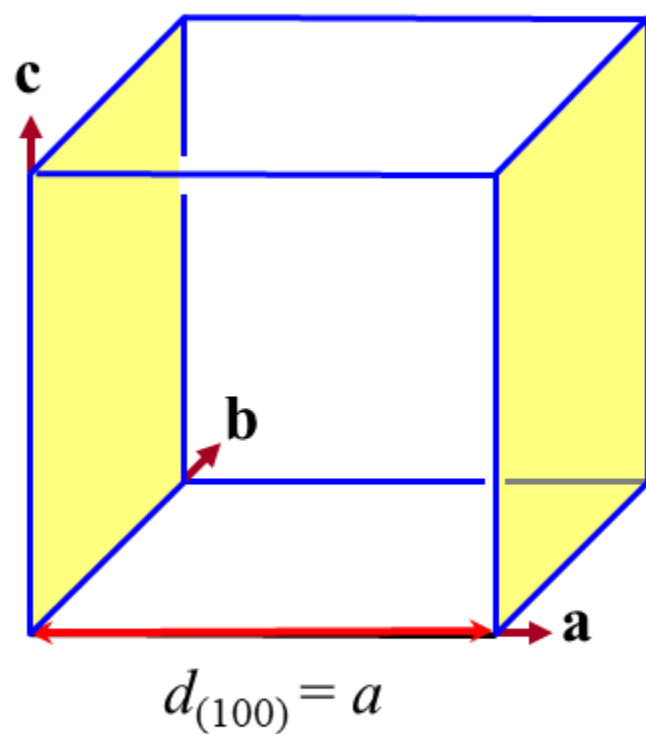
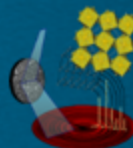
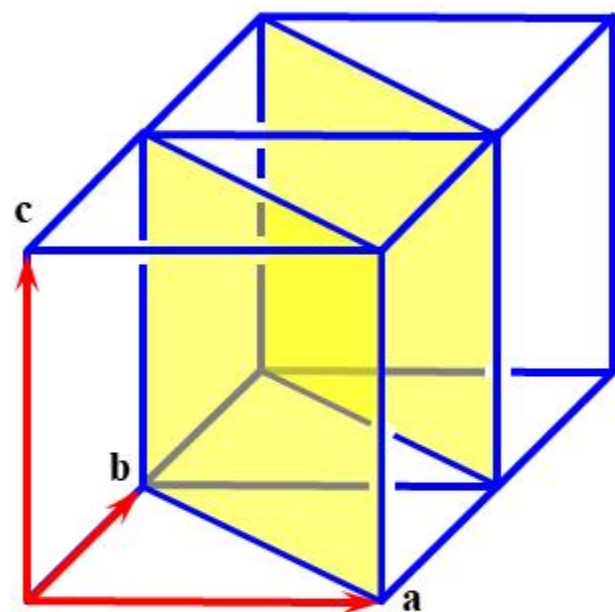
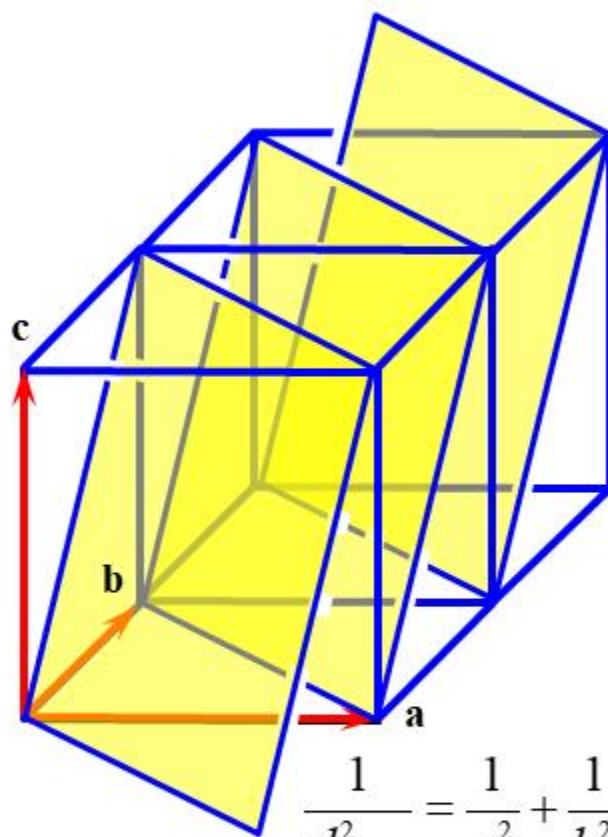


Figure 1.5. Families of (100) and (200) crystallographic planes

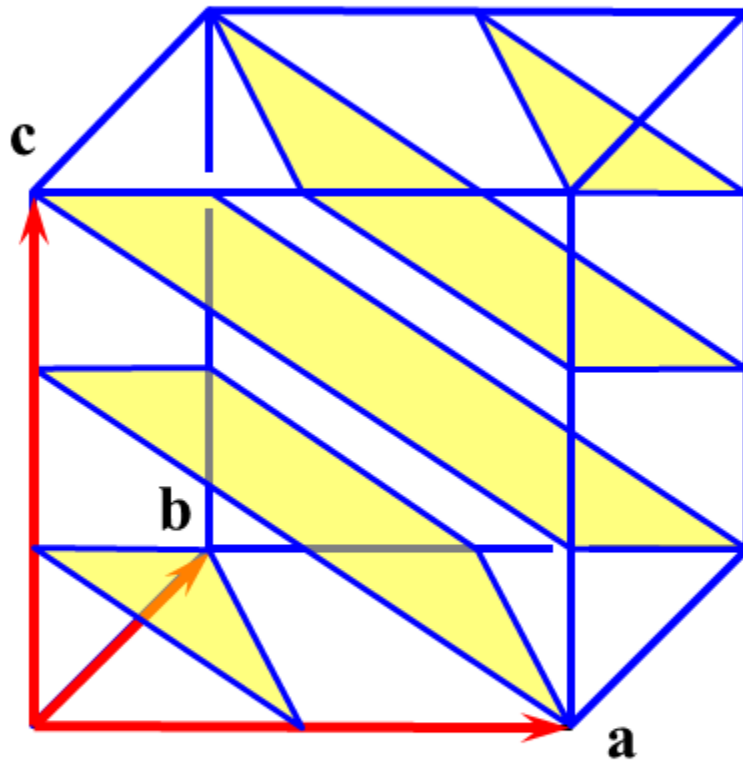


$$\frac{1}{d_{(110)}^2} = \frac{1}{a^2} + \frac{1}{b^2}$$



$$\frac{1}{d_{(\bar{1}\bar{1}1)}^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

Figure 1.6. Families of (110) and $(\bar{1}\bar{1}1)$ crystallographic planes.



$$\frac{1}{d_{(213)}^2} = \frac{4}{a^2} + \frac{1}{b^2} + \frac{9}{c^2}$$

$$\text{Eq. 1.2.} \quad \frac{1}{d_{hkl}} = \sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}$$

Figure 1.7. Family of (213) crystallographic planes.

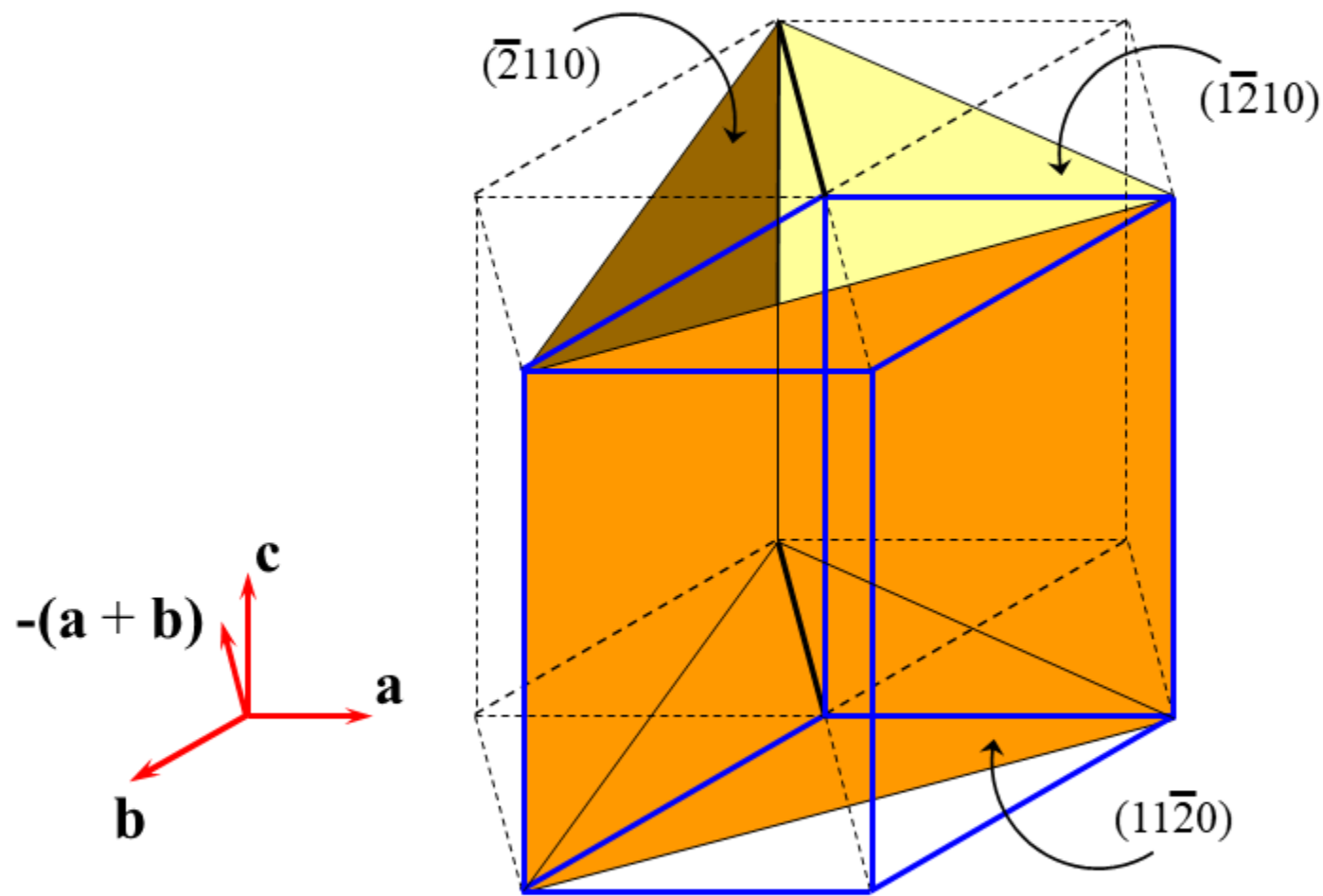
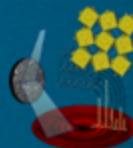


Figure 1.8. Three possibilities to select the crystallographic basis in hexagonal and trigonal crystal systems and the family of $(11\bar{2}0)$ crystallographic planes when $\gamma = 120^\circ$. The indices are shown for the unit cell based on the vectors **a**, **b** and **c**. Three additional symmetrically related families of planes have indices $(\bar{1}\bar{1}20)$, $(\bar{1}2\bar{1}0)$ and $(2\bar{1}\bar{1}0)$ in the same basis and we leave their identification to the reader.

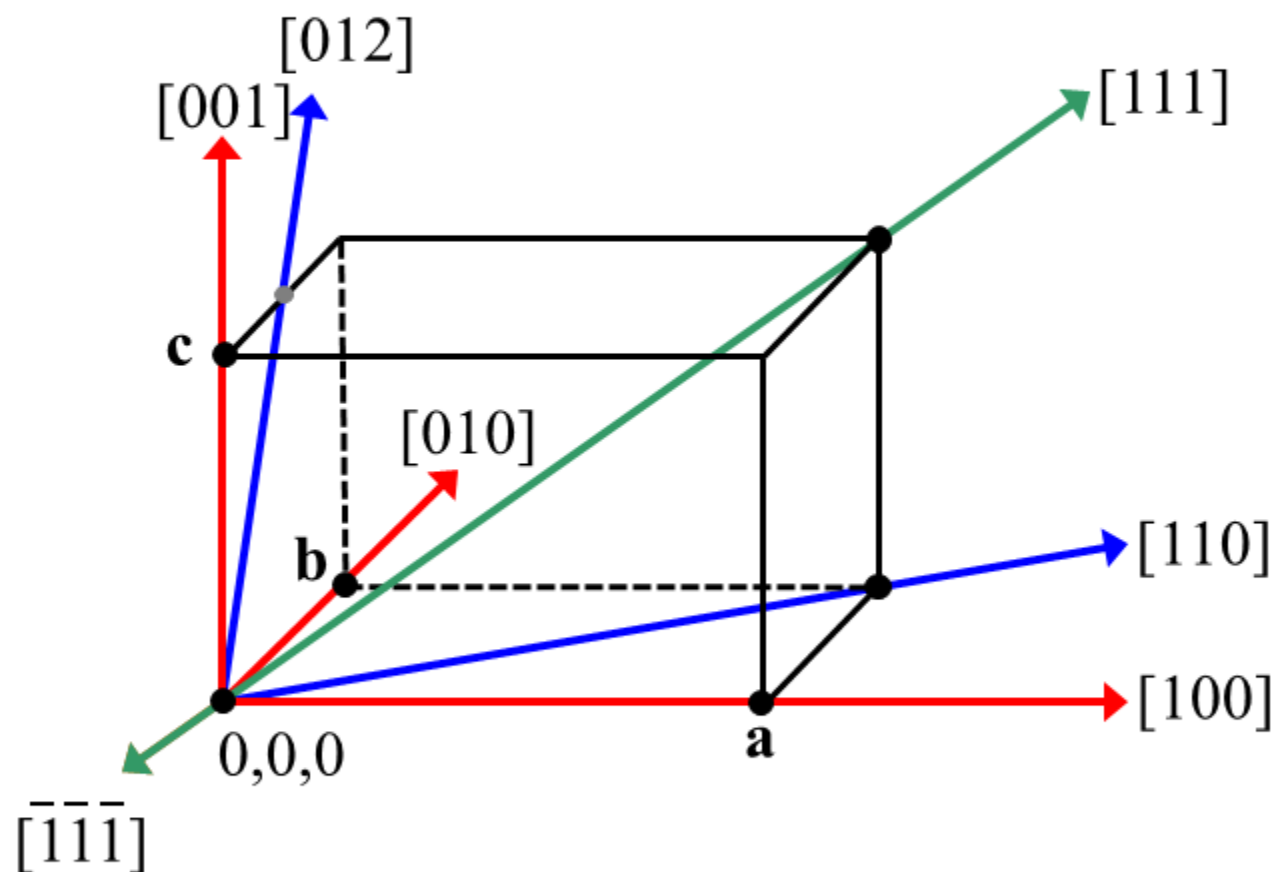
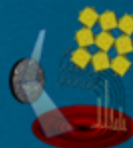
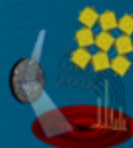


Figure 1.9. Selected crystallographic directions in the lattice with $\alpha = \beta = \gamma = 90^\circ$.



$$\mathbf{v}_3 = \mathbf{v}_1 \times \mathbf{v}_2$$

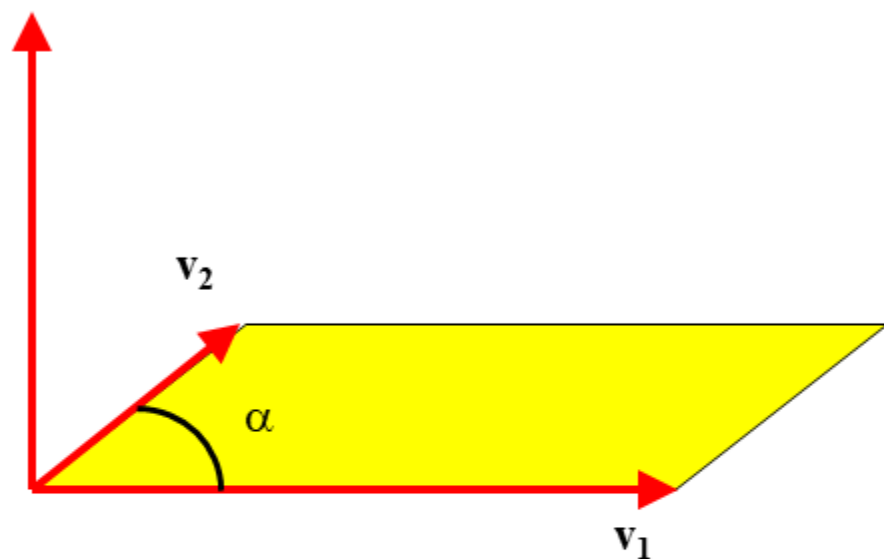


Figure 1.10. Vector (*cross*) product of two vectors. The direction of $\mathbf{v}_3$ is determined using the right-hand rule (thumb of the right hand is aligned with $\mathbf{v}_1$, index finger with $\mathbf{v}_2$, then $\mathbf{v}_3$ is aligned with the middle finger; tails of all vectors face the middle of the palm) or the right-hand screw rule (the direction in which a screw advances when turned from $\mathbf{v}_1$ to $\mathbf{v}_2$).

$$\text{Eq. 1.3. } \mathbf{a}^* \cdot \mathbf{b} = \mathbf{a}^* \cdot \mathbf{c} = \mathbf{b}^* \cdot \mathbf{a} = \mathbf{b}^* \cdot \mathbf{c} \\ = \mathbf{c}^* \cdot \mathbf{a} = \mathbf{c}^* \cdot \mathbf{b} = 0$$

$$\text{Eq. 1.4. } \mathbf{a}^* \cdot \mathbf{a} = \mathbf{b}^* \cdot \mathbf{b} = \mathbf{c}^* \cdot \mathbf{c} = 1$$

$$\text{Eq. 1.5. } \mathbf{v}_1 \cdot \mathbf{v}_2 = v_1 v_2 \cos \alpha$$

$$\text{Eq. 1.6. } |\mathbf{v}_1 \times \mathbf{v}_2| = v_3 = v_1 v_2 \sin \alpha$$

$$\text{Eq. 1.7. } \mathbf{a}^* = \frac{\mathbf{b} \times \mathbf{c}}{V}, \quad \mathbf{b}^* = \frac{\mathbf{c} \times \mathbf{a}}{V}, \quad \mathbf{c}^* = \frac{\mathbf{a} \times \mathbf{b}}{V}$$

$$\text{Eq. 1.8. } \mathbf{a} = \frac{\mathbf{b}^* \times \mathbf{c}^*}{V^*}, \quad \mathbf{b} = \frac{\mathbf{c}^* \times \mathbf{a}^*}{V^*}, \quad \mathbf{c} = \frac{\mathbf{a}^* \times \mathbf{b}^*}{V^*}$$

$$\text{Eq. 1.9. } d_{hkl}^* = \frac{1}{d_{hkl}} \quad \mathbf{d}_{hkl}^* = h\mathbf{a}^* + k\mathbf{b}^* + l\mathbf{c}^*$$

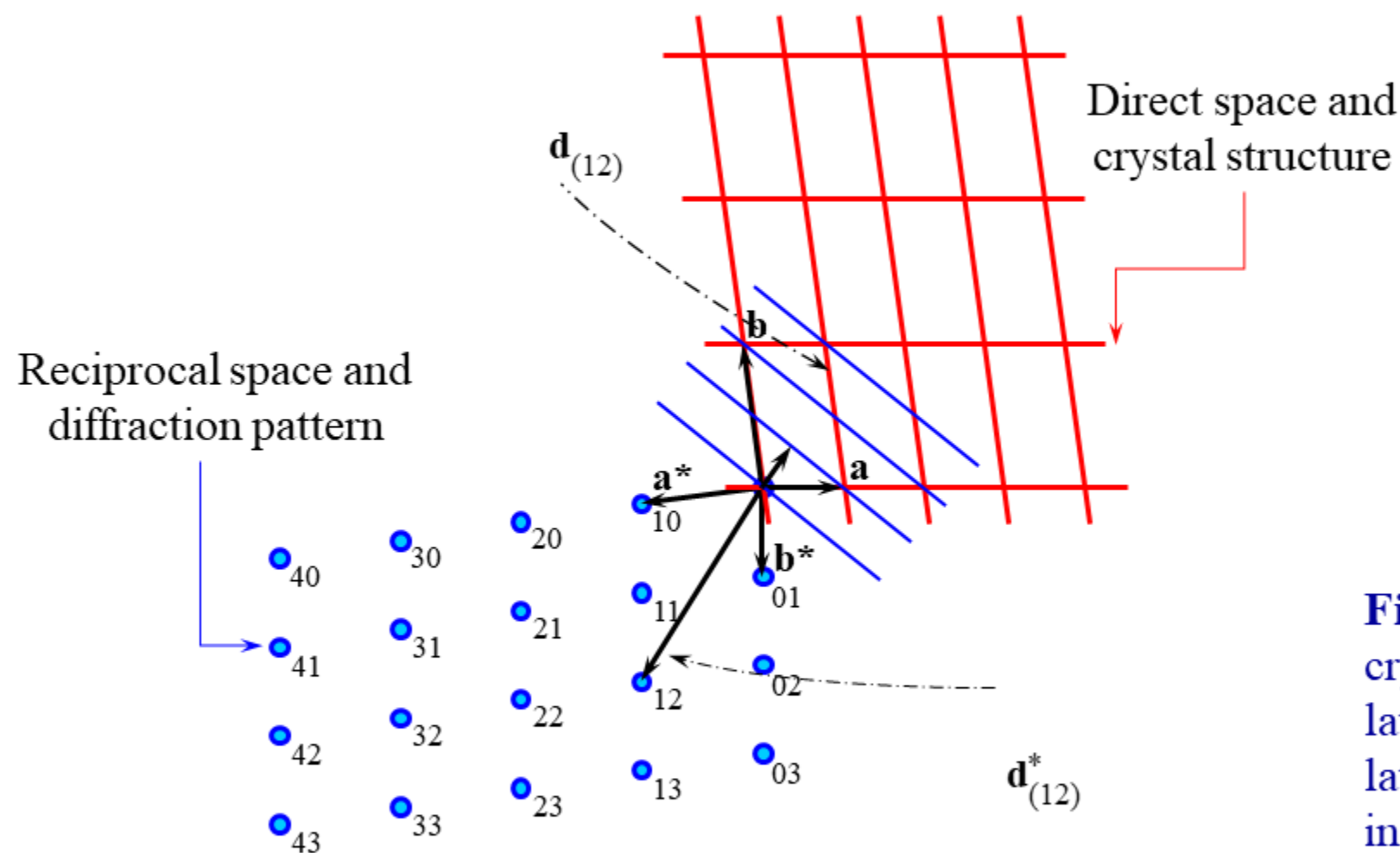
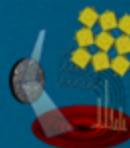


Figure 1.11. Example of converting crystallographic planes in the direct lattice into points in the reciprocal lattice. The corresponding Miller indices are shown near the points in the reciprocal lattice.