

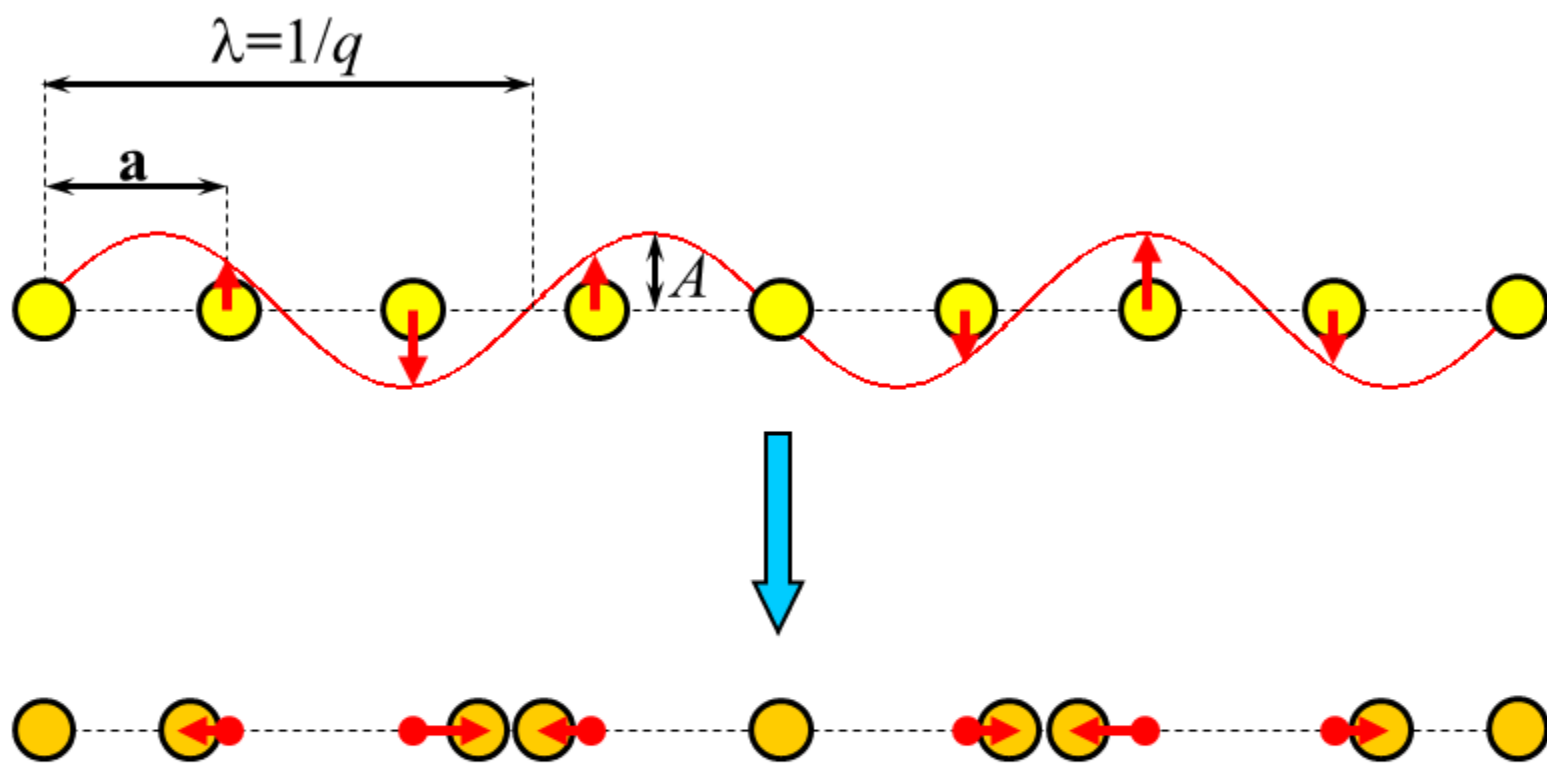
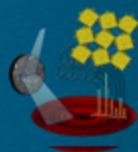


Figure 5.1. The conventional, ideally periodic one-dimensional structure (*yellow circles*) and the corresponding modulation function with the period $\lambda = 1/q$, which is commensurate with $\mathbf{a}$, and the amplitude A (*top*). The resulting commensurately modulated structure (*bottom*).

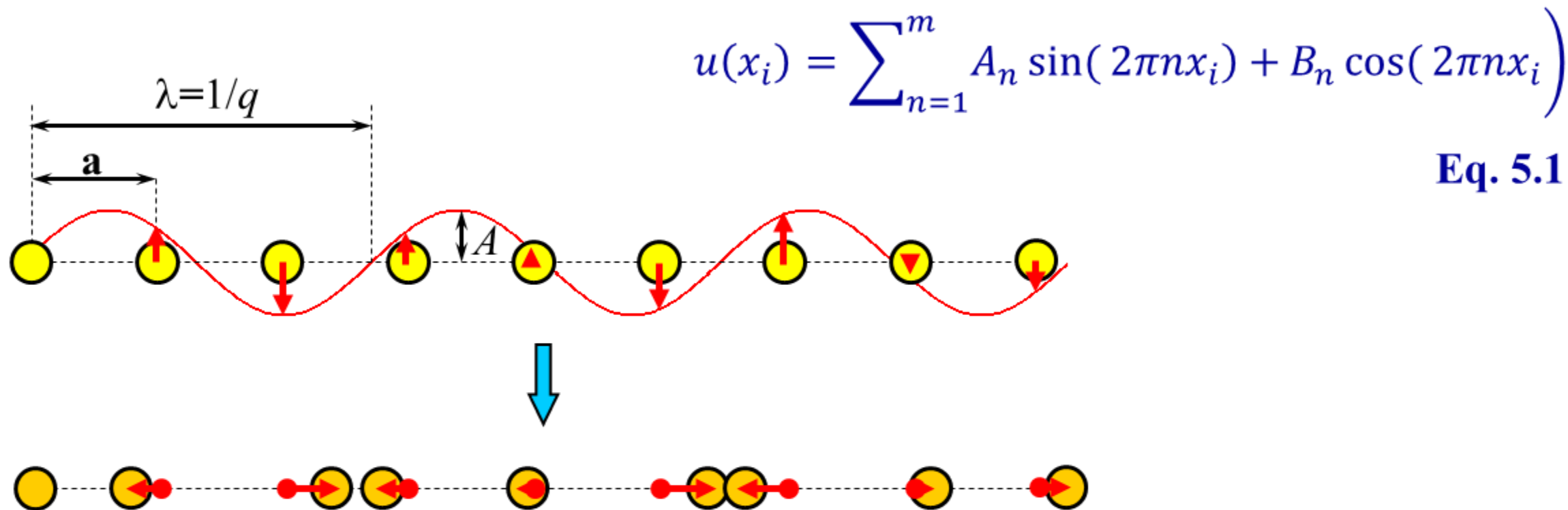
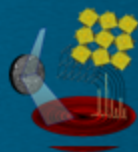
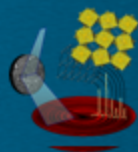


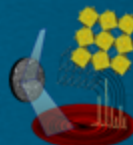
Figure 5.2. The ideally periodic one-dimensional structure (yellow circles) and the corresponding modulation function (red line) with the period $\lambda = 1/q$, which is incommensurate with a , and the amplitude A (top). The resulting incommensurately modulated structure (bottom).



The general form of a superspace symmetry operation may be given as follows:

$$\begin{pmatrix} R_E & 0 \\ R_M & R_I \end{pmatrix} \begin{pmatrix} t_E \\ t_I \end{pmatrix} \quad \text{Eq. 5.2}$$

where: R is rotational component,
 t is translational component,
 E refers to the external (real space) dimensions,
 I refers to the internal (additional) dimension,
 M is their common part.



Two different notations are commonly used at present:

1) the two-line symbol, for example,*

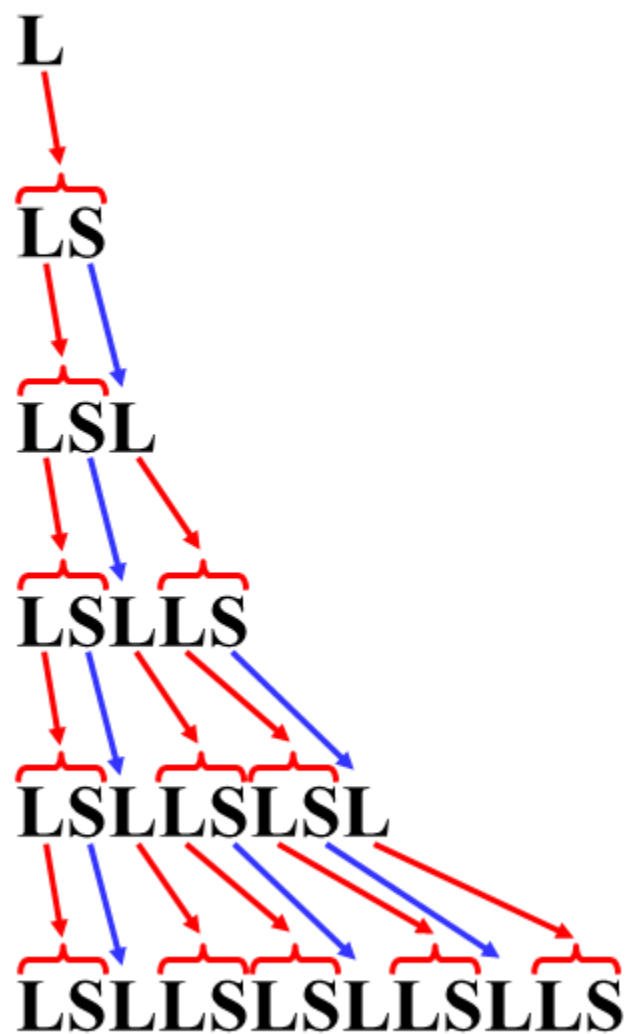
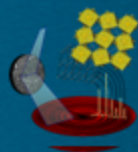
$$B_{s1\bar{1}}^{Pmna} \quad \text{Eq. 5.3}$$

2) the one-line symbol, for example,

$$Pmna(0^{1/2}\gamma)s00 \quad \text{Eq. 5.4}$$

* the two-line symbol written in a single line as:

$$B:Pmna:s1\bar{1}$$



Eq. 5.8.
$$\tau = \lim_{n \rightarrow \infty} \left(\frac{F_{n+1}}{F_n} \right) = \frac{\sqrt{5} + 1}{2} = 1.618\dots$$

The letters L and S are identical to the consecutive members (F_{n+1} and F_n) of the Fibonacci series.

Figure 5.3. The sequence of words containing quasiperiodic sequences of letters L and S based on the following substitution rule: $S \rightarrow L$, and $L \rightarrow LS$.

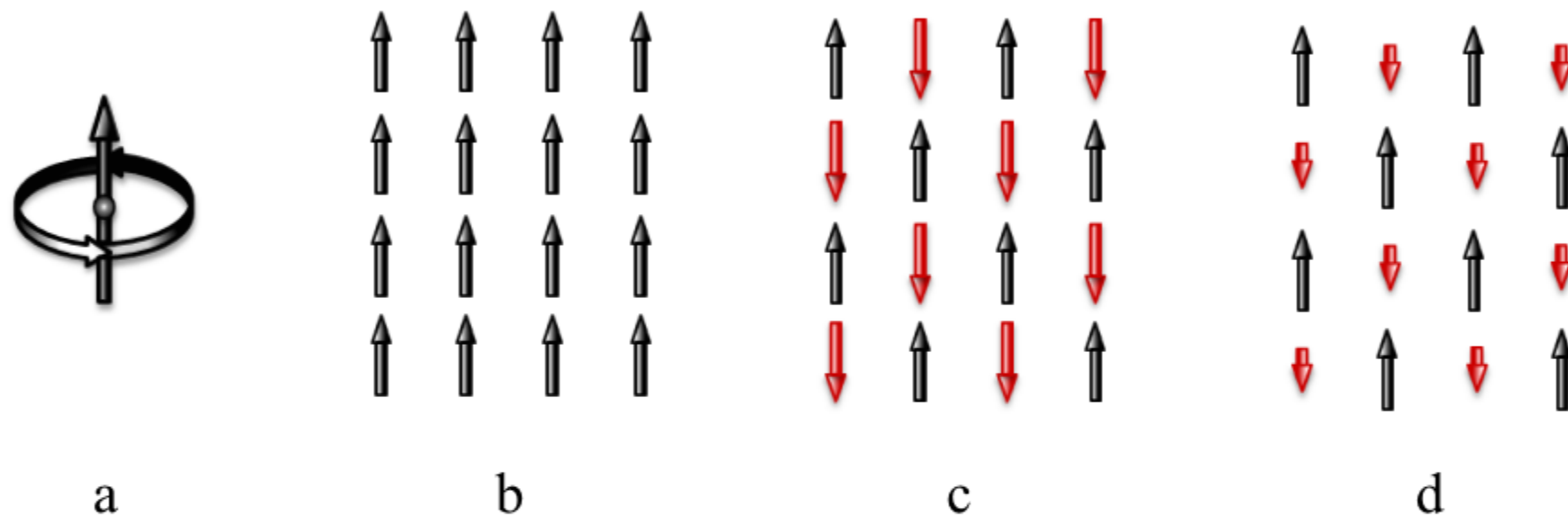
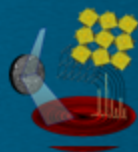


Figure 5.4. Magnetic spin and the resulting magnetic moment are illustrated in (a), while the arrangement of magnetic moments is shown for ferromagnetic (b), antiferromagnetic (c), and ferrimagnetic (d) structures.

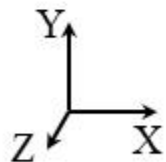
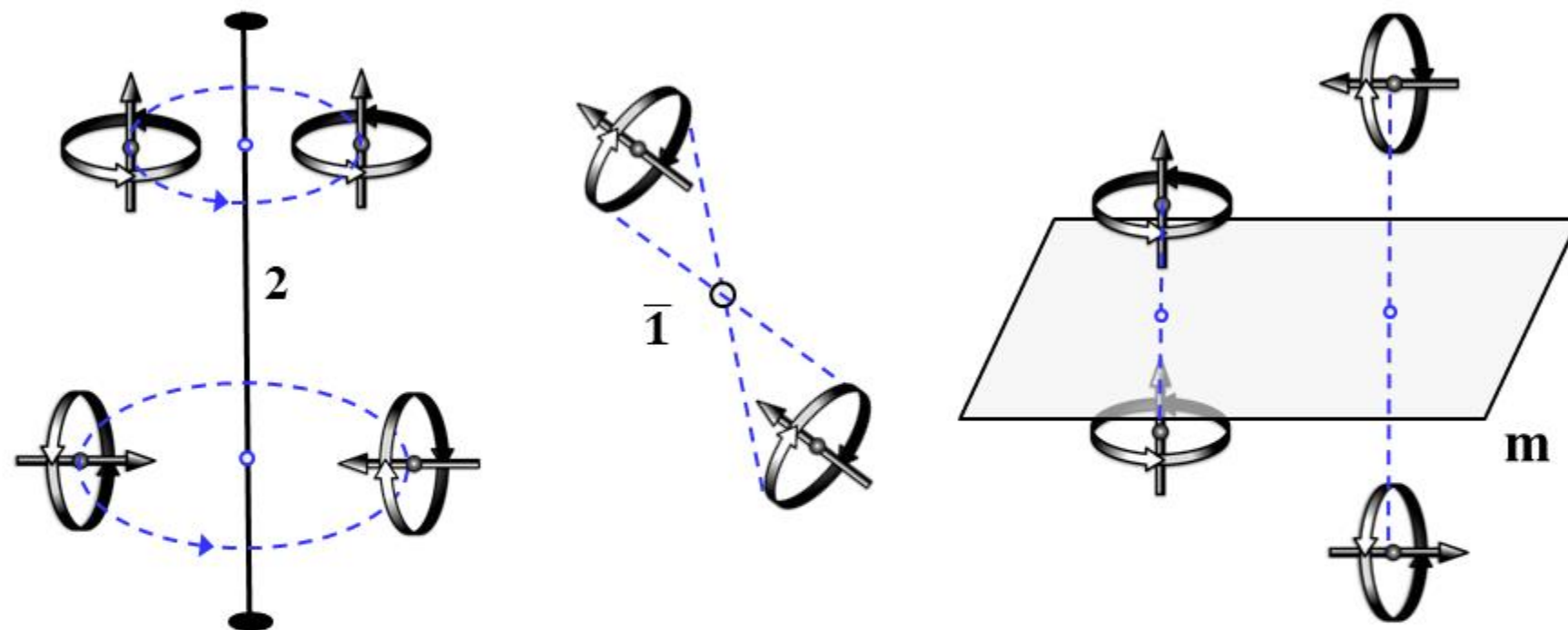


Figure 5.5. Transformation of magnetic spins by a twofold rotation axis (2_y), inversion center ($\bar{1}$), and mirror plane (m_y). The spin is represented as a circle with an arrow pointing in the spinning direction, while the magnetic moment is shown as a vector perpendicular to the circle.

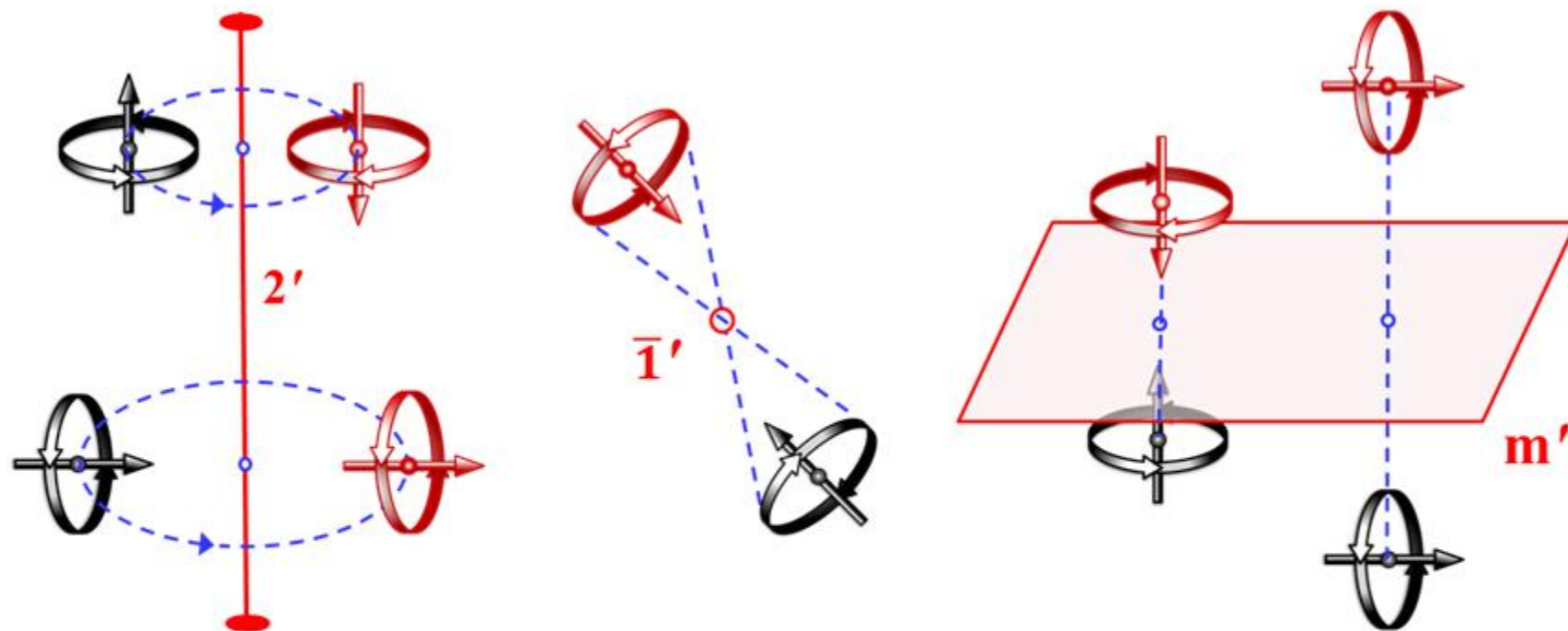


Figure 5.6. Transformation of magnetic spins by a twofold antirotation axis ($2'_y$), antiinversion center ($\bar{1}'$), and antimirror plane (m'_y). The antisymmetry elements, as along with the reversed spins and moments, are shown in red.

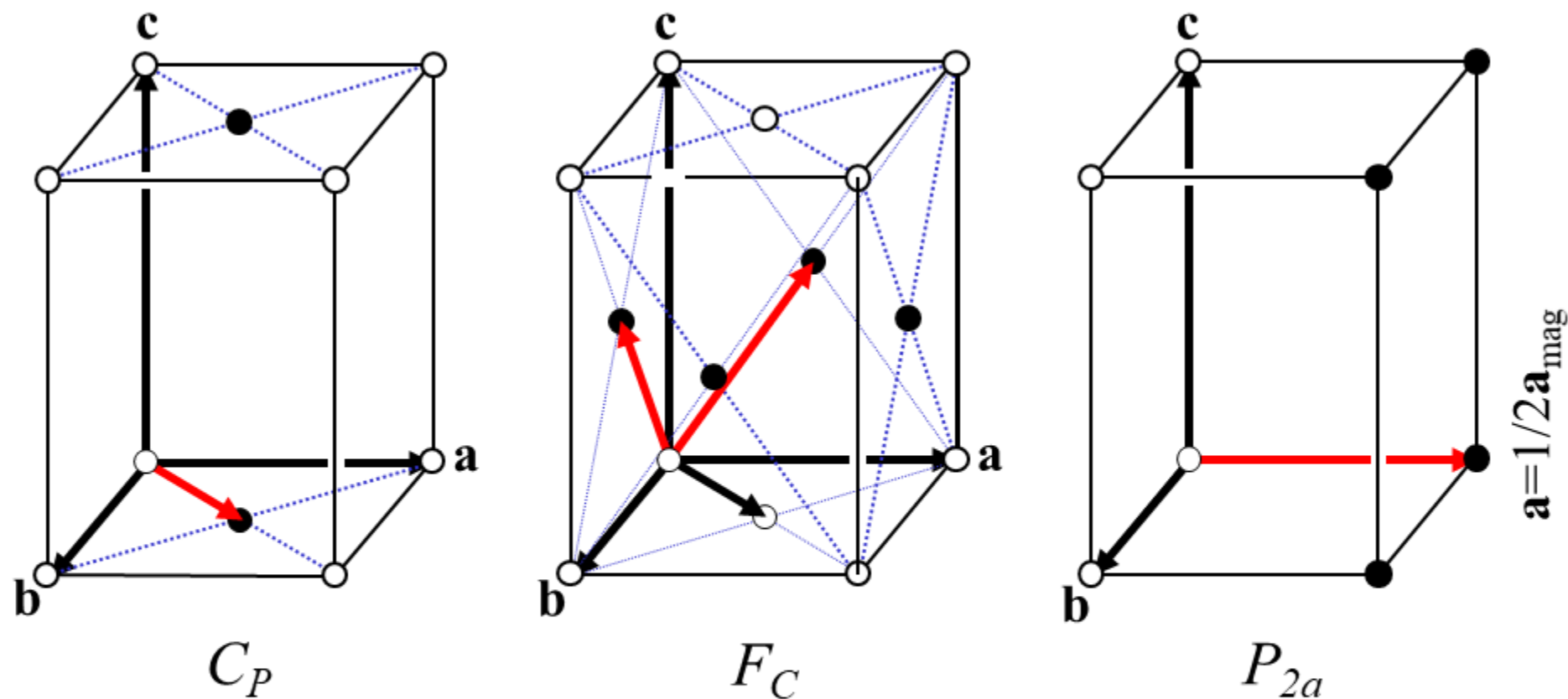
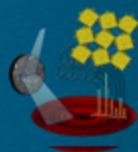
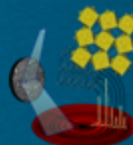


Figure 5.7. Examples of magnetic Bravais's lattices with anti-translation vectors shown in red (C_P : $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$, F_C : $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$, $\frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c}$, and P_{2a} : $\mathbf{a}$) and lattice points with reversed spin shown in black.



Crystal System	Lattice Type
<i>Triclinic</i>	P, P_{2s}
<i>Monoclinic</i>	P, P_{2a} , P_{2b} , P_C C, C_{2c} , C_P
<i>Orthorhombic</i>	P, P_{2a} , P_C , P_F C, C_{2c} , C_P , C_I F, F_C I, I_P
<i>Tetragonal</i>	P, P_{2c} , P_C , P_I I, I_P
<i>Trigonal</i>	R, R_R
<i>Hexagonal</i>	P, P_{2c}
<i>Cubic</i>	P, P_F F I, I_P

Table 5.3. 36 magnetic Bravais lattices (45 lattices counting alternative orientations).

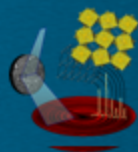


Table 5.4. Three types of magnetic space groups^[1]

Type I		230 uncolored space groups (no antisymmetry)
Type II		230 grey space groups containing the individual anti-identity operation
Type III	(a)	674 black-and-white space groups of the first kind, with primitive magnetic cell = primitive structural cell
	(b)	517 black-and-white groups of the second kind, containing anti-translation with primitive magnetic cell > primitive structural cell
Total		1651 magnetic space groups

^[1] B.D. Litvin, Magnetic Group Tables. 1-, 2- and 3-Dimensional Magnetic Subperiodic Groups and Magnetic Space Groups. Part 1. Introduction (2013). IUCr e-book, ISBN 978-0-9553602-2-0. <https://doi.org/10.1107/9780955360220001>.

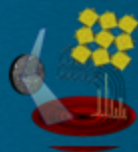


Table 5.5. Comparing symmetry operations for the general position in the $P2_1/m$ space group and its derivatives with different types of magnetic symmetry. Antisymmetry operations are shown in red, with anti-identity and anti-translation operations highlighted in bold.

	$P2_1/m$		$P2_1/m1'$		$P2_1/m'$		$P_{2a}2_1/m$	
	Type I (uncolored)		Type II (grey)		Type IIIa (same cell)		Type IIIb (anti-translation)	
1	$x, y, z, +1$	$[uvw]$	$x, y, z, +1$	$[0, 0, 0]$	$x, y, z, +1$	$[uvw]$	$x, y, z, +1$	$[uvw]$
2	$\bar{x}, y+1/2, \bar{z}, +1$	$[\bar{u}v\bar{w}]$	$\bar{x}, y+1/2, \bar{z}, +1$	$[0, 0, 0]$	$\bar{x}, y+1/2, \bar{z}, +1$	$[\bar{u}v\bar{w}]$	$\bar{x}, y+1/2, \bar{z}, +1$	$[\bar{u}v\bar{w}]$
3	$\bar{x}, \bar{y}, \bar{z}, +1$	$[uvw]$	$\bar{x}, \bar{y}, \bar{z}, +1$	$[0, 0, 0]$	$\bar{x}, \bar{y}, \bar{z}, -1$	$[\bar{u}v\bar{w}]$	$\bar{x}, \bar{y}, \bar{z}, +1$	$[uvw]$
4	$x, \bar{y}+1/2, z, +1$	$[\bar{u}v\bar{w}]$	$x, \bar{y}+1/2, z, +1$	$[0, 0, 0]$	$x, \bar{y}+1/2, z, -1$	$[u\bar{v}w]$	$x, \bar{y}+1/2, z, +1$	$[\bar{u}v\bar{w}]$
5			$x, y, z, -1$	$[0, 0, 0]$			$x+1, y, z, -1$	$[\bar{u}v\bar{w}]$
6			$\bar{x}, y+1/2, \bar{z}, -1$	$[0, 0, 0]$			$\bar{x}+1, y+1/2, \bar{z}, -1$	$[u\bar{v}w]$
7			$\bar{x}, \bar{y}, \bar{z}, -1$	$[0, 0, 0]$			$\bar{x}+1, \bar{y}, \bar{z}, -1$	$[\bar{u}v\bar{w}]$
8			$x, \bar{y}+1/2, z, -1$	$[0, 0, 0]$			$x+1, \bar{y}+1/2, z, -1$	$[u\bar{v}w]$

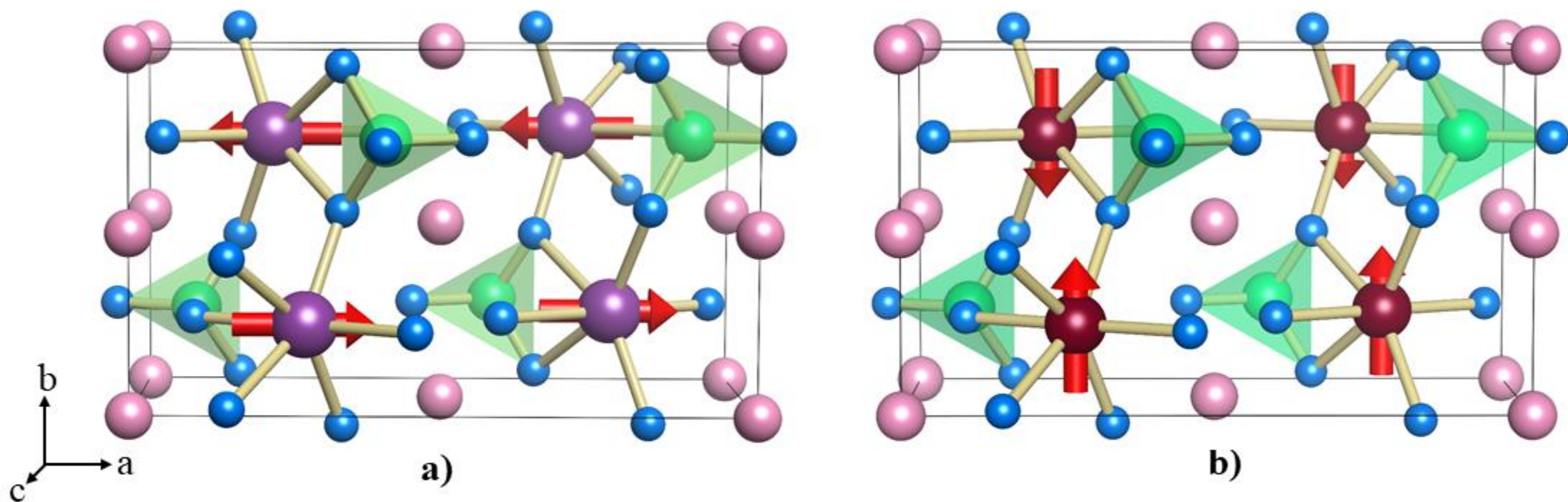


Figure 5.8. Orientation of magnetic vectors in (a) $Pn'm'a'$ for $LiMnPO_4$ and (b) $Pnma'$ for $LiCoPO_4$. Li atoms are shown in *pink*, P atoms are *green balls*, and O atoms shown as *small blue balls*. Mn and Co atoms are represented by *large purple and dark red balls*, respectively.

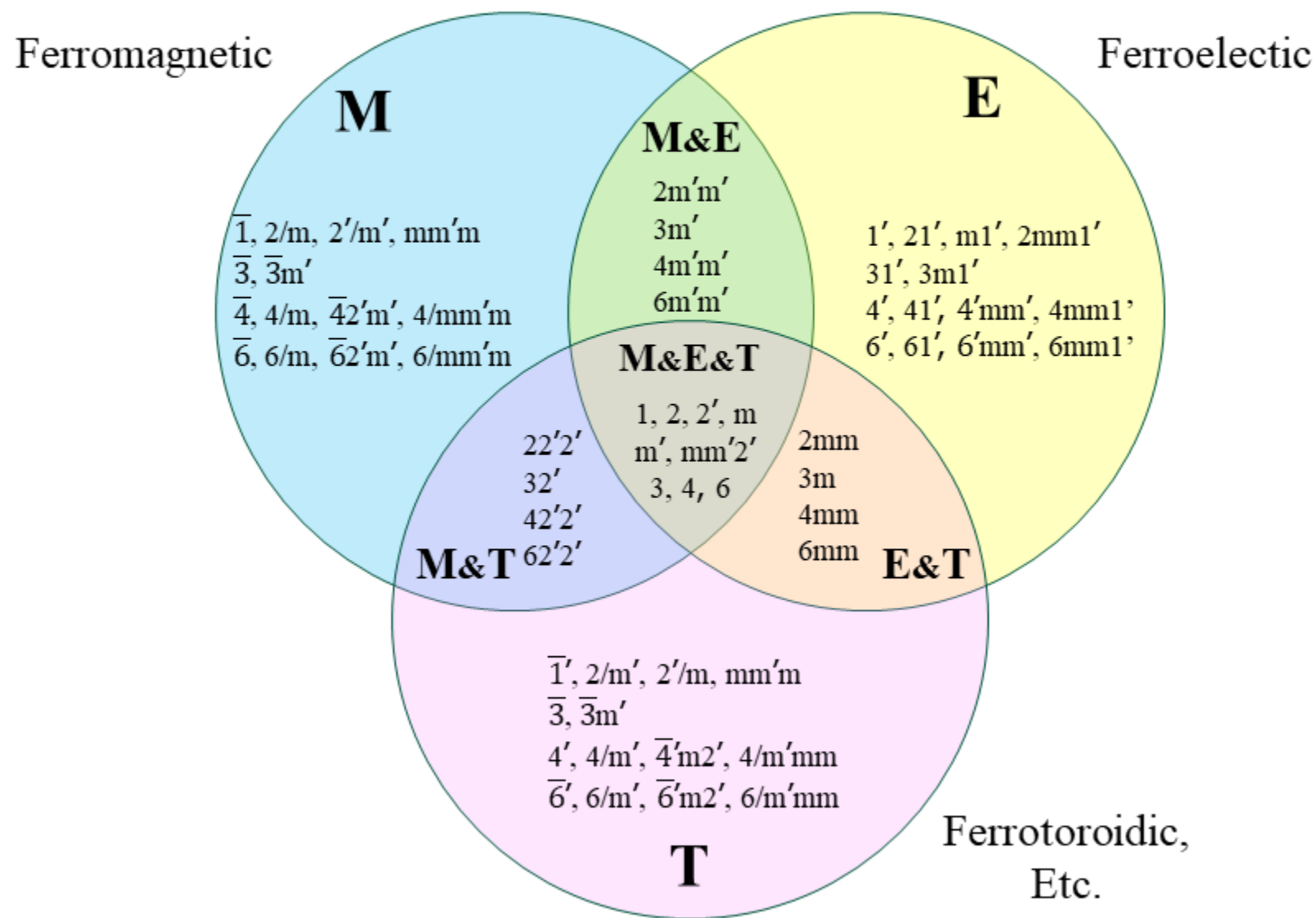
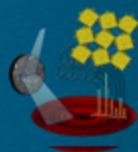


Figure 5.9. Ascher's trinity diagram linking magnetic symmetry to physical properties.